Multivariate P- and/or Q-polynomial association schemes

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The study of P-polynomial association schemes (distance-regular graphs) and Q-polynomial association schemes, and in particular P- and Q-polynomial association schemes, has been a central theme not only in the theory of association schemes but also in the whole study of algebraic combinatorics in general. Leonard's theorem says that the spherical functions of P- and Q-polynomial association schemes are described by Askey-Wilson orthogonal polynomials or their relatives, which are one-variable orthogonal polynomials. It seems that the new attempt to define and study multivariate P- and Q-polynomial association schemes had been hoped for, but had gotten only limited success. Recently, Bernard, Crampé, d'Andecy, Vinet, and Zaimi defined bivariate P- and/or Q-polynomial association schemes [3].

In this talk, we will first introduce our new modified definition for multivariate P- and/or Q-polynomial association schemes with respect to a general monomial order [1]. We will also present several interesting families of multivariate P- and/or Q-polynomial association schemes, including nonbinary Johnson schemes and association schemes from attenuated spaces [2].

This is joint work with Eiichi Bannai, Hirotake Kurihara and Da Zhao.

References

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