Construction of spherical designs and unitary designs

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The existence of spherical designs for arbitrary dimension d and strength t is proved by Seymour–Zaslavsky [1] in 1980s. A bound on the size of spherical designs is given by Bondarenko–Radchenko–Viazovska [2, 3] in last decade. Recently Xiang [4] provides the first explicit construction of spherical designs. The existence of unitary designs follows from Seymour–Zaslavsky approach directly. The Clifford groups stand as a family of unitary 3-designs [5]. Bannai–Navarro–Rizo–Tiep summarize the finite groups which serve as unitary t-designs [6]. The first unitary 4-design on U(4) for 2-qubits is given in [7]. We provide an explicit construction of unitary designs for arbitrary dimension d and strength t [8], which gives another explicit construction of spherical designs as a byproduct. Recently we find a new construction of unitary designs reversely from spherical designs.

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