Monster embeddings of 3-transposition groups via Majorana rerpesentations

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The Monster group M is the largest sporadic simple group with more than $8 \cdot 10^{53}$ elements. In addition, it is the group of automorphisms of the $196,884$-dimensional Fischer-Griess algebra V_M, which is equipped with a positive definite inner product $(\cdot, \cdot)$, and a commutative, non-associative algebra $\cdot$, which satisfy to the relation $(x \cdot y, z) = (x, y \cdot z)$. The algebra V_M is generated by a set of axial vectors A. In 2009, A. A. Ivanov axiomatized some properties of the axes $a \in A$ and introduced the notions of Majorana algebra and Majorana representation. Later, Majorana theory proved to be a powerful machinery to study the subgroup structure of M, and the subalgebra structure of V_M.

The 3-transposition groups with a trivial center and simple derived subgroups are categorized by B. Fischer. In addition, the Monster group M contains subgroups isomorphic to quite big 3-transposition groups, or their subgroups of index 2. Therefore, there is a motivation to study Majorana representations of 3-transposition groups.

Firstly, we find the sizes of the maximal symmetric subgroups of the groups from the Fischer list, generated by the transpositions. Then, we use this information to find all pairs of 3-transposition groups from the Fischer list, which can be embedded into each other. Furthermore, we find groups from the Fischer list, which admit a standard Majorana representation. The main result is that a group from the Fischer list, except possibly Fi_{24}, admits a standard Majorana representation if and only if it can be embedded in the Monster group.