On the Packing Density of Lee Spheres

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Based on the packing density of cross-polytopes in $\mathbb{R}^n$, more than 50 years ago Golomb and Welch proved that the packing density of Lee spheres in $\mathbb{Z}^n$ must be strictly smaller than 1 provided that the radius r of the Lee sphere is large enough compared with n where $n > 2$. In the same paper [1], they conjectured that there is no tiling of Lee spheres of radius r in $\mathbb{Z}^n$ for $n \geq 3$ and $r \geq 2$.

In this talk, we investigate the lattice packing density of Lee spheres of fixed radius r for infinitely many n. First we present several methods to prove the nonexistence of (almost) perfect lattice packing of Lee spheres in $\mathbb{Z}^n$. Second, we look at the constructions of lattice packings with density $\delta_n \to \frac{2^r}{(2r+1)r!}$ as $n \to \infty$. When $r = 2$, the packing density can be improved to $\delta_n \to \frac{2}{3}$ as $n \to \infty$.

References

[1] S. W. Golomb and L. R. Welch. Perfect codes in the Lee metric and the packing of polyominoes. *SIAM Journal on Applied Mathematics*, 18(2):302–317, 1970.