Storage Codes on Triangle-Free Graphs

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This is joint work with Haihua Deng, Hexiang Huang, Guobiao Weng.

Abstract: Consider a simple, connected graph Γ with n vertices. Let C be a code of length n with its coordinates corresponding to the vertices of Γ . We define C as a *storage code* on Γ if, for any codeword $c \in C$, the information at each coordinate of c can be recovered by accessing its neighboring coordinates. The main problem here is to construct high-rate storage codes on triangle-free graphs. In this paper, we employ the polynomial method to address a question asked by Barg and Zémor in 2022, demonstrating that the BCH family of storage codes on triangle-free Cayley graphs achieves a unit rate. Furthermore, we generalize the construction of the BCH family and obtain more storage codes of unit rate on triangle-free graphs. We also compare the BCH family with the other known constructions by examining the rate of convergence of $1/(1 - R(C_n))$ with respect to the length n, where $R(C_n)$ is the rate of code C_n . At last, we reveal a connection between the storage codes on triangle-free graphs and the Ramsey number R(3, t), which leads to an upper bound for the rate of convergence of $1/(1 - R(C_n))$.