

Graphs and Groups, Complexity and Convexity (G2C2-2024)

(Summer School, August 11-25, 2024, Hebei Normal University, Shijiazhuang, China)

Title: Invariants of graphs, embedded graphs, delta-matroids, and permutations

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Description:

Theory of graph invariants goes back as far as to the work by L.Euler in XVIII century. The minicourse is devoted to invariants of various objects of combinatorial and topological origin, namely, graphs and framed graphs, knots and links in the 3-sphere, embedded graphs and delta-matroids. An invariant is a function taking the same value on isomorphic objects. In the lectures, we are going to address the problem of extending a given invariant from its original domain to other objects. The lecturer is planning to introduce the participants to most recent results in the field.

No preliminary knowledge is required.

Lecture 1. Graphs and their invariants

Families of graphs. Graph presentations. Graph isomorphism. Examples of graph invariants: chromatic function, degeneracy. Deletion-contraction relation.

Lecture 2. Embedded graphs

Two-dimensional surfaces. Orientability. Graphs on surfaces. Duality. Permutation presentation of a graph on a surface. Number of faces.

Lecture 3. Delta-matroids

Definition of a delta-matroid. Examples. Associating a delta-matroid to a graph and to an embedded graph.

Lecture 4. Invariants of delta-matroids. Partial duality

Number of vertices. Number of faces. Interlace polynomial.

Lecture 5. Invariants of knots

Knots and their invariants. Plane diagram of a knot. Reidemeister moves.

Lecture 6. Singular knots, finite type invariants of knots, and chord diagrams

Vassiliev knot invariants and stratification in the space of knots. Chord diagrams and 4-term relations for them. Weight systems.

Lecture 7. Intersection graphs and 4-term relations for graphs

4-term relations for graphs. Graph and embedded graph invariants satisfying 4-term relations.

Lecture 8. Weight systems associated to Lie algebras

Weight systems associated to Lie algebras. $Sl(2)$ - and $gl(N)$ -weight systems. The universal gl -weight system

Bibliography

Definitely, the bibliography is far from being complete.

Books

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Papers

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