**Minimum number of disjoint pairs in a uniform family of subsets**

Gyula O.H. Katona
*Rényi Institute, Budapest*
ohkatona@renyi.hu

Let $[n] = \{1, 2, \ldots, n\}$ be our underlying set. $\binom{[n]}{k}$ will denote the family of all $k$-element subsets of $[n]$. A family $\mathcal{F} \subset \binom{[n]}{k}$ is called intersecting if any pair of its members have a non-empty intersection. The celebrated theorem of Erdős, Ko and Rado [3] states that if $2k \leq n,\ \mathcal{F} \subset \binom{[n]}{k}$ is intersecting then $|\mathcal{F}| \leq \binom{n-1}{k-1}$.

Therefore if $\mathcal{F}$ is larger, then there is a pair of disjoint members. But what is the minimum of the number $\mathrm{dp}(\mathcal{F})$ of the disjoint pairs if $|\mathcal{F}|$ is given? This question was solved for the case $k = 2$ in [1]. List the characteristic vectors (here two 1's and $n-2$ 0's) of the subsets in a lexicographic order. Then the minimum of $\mathrm{dp}(\mathcal{F})$ is obtained either for the first or for the last $|\mathcal{F}|$ members. For arbitrary $k$ the first result was found in [5] : when $|\mathcal{F}| = \binom{n-1}{k-1} + 1$ then $\mathrm{dp}(\mathcal{F}) \geq \binom{n-k-1}{k-1}$ and the corresponding construction consists of the lexicographically first members. A major step in this direction was the result of Das, Gan and Sudakov [2]: when $|\mathcal{F}| = \binom{n-1}{k-1} + r$ and $n$ is large enough with respect to $r$ then $\mathrm{dp}(\mathcal{F})$ is maximized for the lexicographically first members, again.

Let $\mathrm{DP}(\mathcal{F})$ denote the graph where the vertices are the members of $\mathcal{F}$ and two such vertices are adjacent if the corresponding sets are disjoint. If $|\mathcal{F}| = \binom{n-1}{k-1} + 1$ then $\mathrm{dp}(\mathcal{F}) = \binom{n-k-1}{k-1}$ achieved for a construction in which the edges of $\mathrm{DP}(\mathcal{F})$ form a star. In [4] we proved that excluding this possibility, the "second best value"of $\mathrm{dp}(\mathcal{F})$ is $2\left(\binom{n-k-1}{k-1} - 1\right)$. The number of disjoint pairs is almost the double. In general we give a good lower bound on the number of disjoint pairs when $\tau(\mathrm{DP}(\mathcal{F}))$, the minimum number of vertices covering all edges of $\tau(\mathrm{DP}(\mathcal{F}))$ is given.

**References**

[1] R. Ahlswede, G.O.H. Katona, Graphs with maximal number of adjacent pairs of edges, *Acta Math. Acad. Sci. Hungar.* **32**(1978) 97-120.

[2] S. Das, W. Gan, B. Sudakov, The minimum number of disjoint pairs in set systems and related problems, *Combinatorica* **36**(6)(2016) 623 – 660.

[3] P. Erdős, Chao Ko and R. Rado, Intersection theorems for systems of finite sets, *Q. J. Math. Oxf. II Ser.* **12**(1961) 313-318.

[4] Joanna Jasińska, Gyula O.H. Katona, The number of disjont pairs in families of $k$-element subsets, submitted.

[5] G.O.H. Katona, G.Y. Katona and Z. Katona, Mostprobably intersecting families of subsets, *Combinatorics, Probability and Computing* **21**(1-2)(2012) 219 – 227. .