The Thompson group: opening a path to modern group theory

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We enhance the Thompson–Smith construction [5–8] of the Thompson sporadic simple group E. As a result, (a) we obtain a conceptual explanation of the dichotomy between Lie and finite cases in terms of representations on an $(L_2(8) : 3)$-subgroup; (b) along with $2^5 \cdot L_5(2)$ and $2^{1+8}_+ \cdot A_9$, we construct in E subgroups $^3D_4(2) : 3$, $(F_{21} \times L_3(2)) : 2$, $(G_2(3) \times 3) : 2$ and $U_3(8) : 6$; (c) we consider the coset geometry of the above subgroups and identify the corresponding geometric presentation with that by Havas–Soicher–Wilson [4]; (d) this amounts to a geometric construction and uniqueness proof for E (the uniqueness issue for the Thompson group was listed [1] as one of just two outstanding in the Classification of Finite Simple Groups); (e) we deliver an elementary self-contained construction of the Dempwolff–Thompson orthogonal decomposition on the E_8-Lie algebra, along with the stabiliser $2^{5+10} \cdot L_5(2)$ of this decomposition in the Lie group $E_8(\mathbf{C})$. The work was further stimulated by the recent reappearance of E in the Moonshine context [2, 3].

References

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