On Boolean degree 1 functions (Cameron-Liebler sets) in finite vector spaces

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It is easy to see that if f is a real, n-variate affine function which is Boolean on the n-dimensional hypercube (that is, $f(x) \in \{0,1\}$ for $x \in \{0,1\}^n$), then $f(x) = 0$, $f(x) = 1$, $f(x) = x_i$ or $f(x) = 1 - x_i$. The same classification holds if we restrict $\{0,1\}^n$ to elements with Hamming weight k if $n - k, k \geq 2$.

Let $V(n, q)$ denote the n-dimensional vector space over the field with q elements. Since work by Cameron and Liebler in 1982, it has been asked if a similar classification holds for k-spaces in $V(n, q)$. It is known due to the work by Drudge (1998) and subsequent work that for $(n, k) = (4, 2)$ such a classification is impossible. In our talk we will discuss the history of the problem. Furthermore, we will show that for fixed $q, k \geq 2$ and n sufficiently large, a Boolean degree 1 function on the k-spaces of $V(n, q)$ corresponds to one of the following:

1. The empty set.
2. All k-spaces through a fixed 1-space P.
3. All k-spaces in a fixed hyperplane H.
4. The union of the previous two examples when P is not in H.
5. The complement of any of the previous cases.

The solves the classification problem of Cameron-Liebler classes asymptotically.