Divisible design graphs from symplectic graphs

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This talk is based on joint work with Sergey Goryainov, Willem Haemers and Leonid Shalaginov [1].

A regular graph of degree k on v vertices is called a *divisible design graph* with parameters $(v, k, \lambda_1, \lambda_2, m, n)$ if its vertex-set can be partitioned into m classes of size n such that any two vertices from the same class have λ_1 common neighbours and any two vertices from different classes have λ_2 common neighbours. Divisible design graphs were introduced in [2] in view of their connection with divisible designs: the adjacency matrix of any such graph is an incidence matrix of such a design.

In the talk, new families of divisible design graphs are constructed that are related to the symplectic graphs $Sp(2e, q)$, $e \geq 2$. Starting from a $2e$-dimensional vector space V over the finite field $\mathbb{F}_q$ that is endowed with a nondegenerate alternating bilinear form $b(\cdot, \cdot)$, the vertices of $Sp(2e, q)$ are the one-dimensional subspaces of V, where two distinct one-dimensional subspaces $\langle \bar{v}_1 \rangle$ and $\langle \bar{v}_2 \rangle$ are adjacent whenever $b(\bar{v}_1, \bar{v}_2) = 0$.

We define and discuss a family of divisible design graphs based on a partition of $Sp(4, q)$, q odd, in subgraphs isomorphic to $K_{q+1,q+1}$, and show that there is an example in this family for every odd prime power q. We have classified by computer all examples in this family for $q \in \{3, 5, 7\}$ and we discuss the computational challenges that we faced during this process.

The divisible design graphs in this family also give rise to additional examples of divisible design graphs. Finally, we also describe some families of divisible design graphs based on so-called symplectic spreads of $Sp(2e, q)$.

References

[1] B. De Bruyn, S. Goryainov, W. H. Haemers and L. Shalaginov, Divisible design graphs from the symplectic graph. https://arxiv.org/abs/2404.09902

[2] W. H. Haemers, H. Kharaghani and M. A. Meulenberg, Divisible design graphs. *J. Combin. Theory Ser. A* **118** (2011), 978–992.