First-order model theory, surjunctivity, and Kaplansky's stable finiteness conjecture

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A ring R is *directly finite* if $ab = 1$ implies $ba = 1$ for all $a, b \in R$. A ring R is *stably finite* if the ring $\mathrm{Mat}_d(R)$ of $d \times d$ matrices with entries in R is stably finite for every integer $d \geq 1$. A group G is *surjunctive* if for any finite alphabet set A, every injective cellular automaton $\tau \colon A^G \to A^G$ is surjective. Using algebraic geometry methods, Xuan Kien Phung [5] proved that the group ring $K[G]$ of a surjunctive group G with coefficients in a field K is always stably finite. In other words, every group satisfying *Gottschalk's conjecture* also satisfies *Kaplansky's stable finiteness conjecture*. Based on a joint work with Michel Coornaert and Phung [3], I'll present a proof of this result based on first-order Model Theory. (For related background material, see [1,2] for Geometric Group Theory and Symbolic Dynamics, and [4] for Model Theory).

References

[1] Tullio Ceccherini-Silberstein and Michel Coornaert, *Cellular Automata and Groups*, Springer Monographs in Mathematics, Springer-Verlag, Berlin, 2010.

[2] Tullio Ceccherini-Silberstein and Michel Coornaert, *Exercises in Cellular Automata and Groups*, Springer Monographs in Mathematics, Springer, Cham, 2023.

[3] Tullio Ceccherini-Silberstein, Michel Coornaert, and Xuan Kien Phung, First-order model theory and Kaplansky's stable finiteness conjecture. Preprint arXiv:2310.09451.

[4] David Marker, *Model theory*, vol. 217 of Graduate Texts in Mathematics, Springer-Verlag, New York, 2002.

[5] Xuan Kien Phung, A geometric generalization of Kaplansky's direct finiteness conjecture. *Proc. Amer. Math. Soc.* **151** (2023), no. 7, 2863-2871.